Quantum Computation

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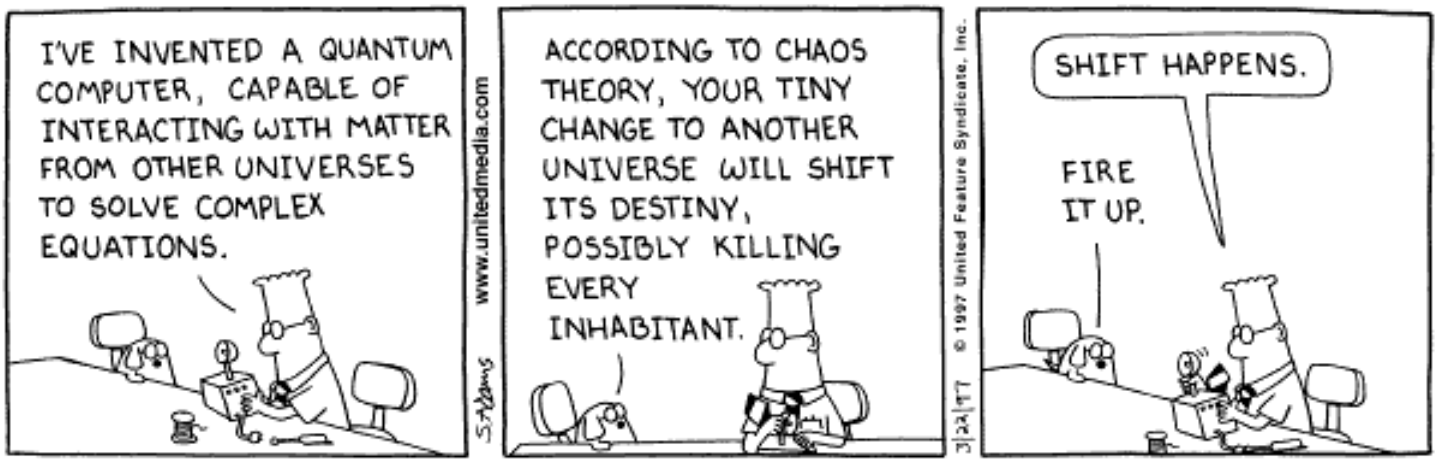


What does it mean to compute?

Q: Is there a general algorithm to determine whether a mathematical conjecture is true or false?

Church-Turing: **NO!**

Church-Turing thesis: Any algorithmic process can be simulated on a Turing machine.

Ad hoc empirical justification.

Strong Church-Turing thesis: Any algorithmic process can be efficiently simulated on a probabilistic Turing machine.

Deutsch:

Can we justify C-T thesis using laws of physics?

Quantum mechanics seems to be very hard to simulate on a classical computer.

Might it be that computers exploiting quantum mechanics are not efficiently simulatable on a probabilistic Turing machine?

(Violation of strong C-T thesis!)

Might it be that such a computer can solve some computational problems faster than a probabilistic Turing machine?

Candidate universal computer: quantum computer

Quantum circuit model

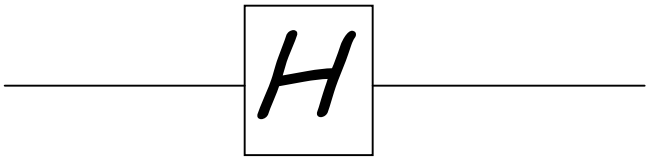
Classical	Quantum
Unit: bit	Unit: qubit
1. Prepare n bit input	1. Prepare n qubit input
2. Logic gates	2. Reversible quantum logic gates
3. Readout value of bits	3. Readout partial information about qubits

Single qubit quantum logic gates

Pauli gates:

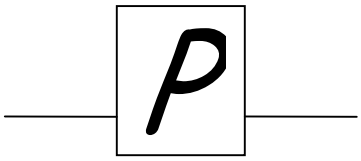
$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}; \quad Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}; \quad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Hadamard gate:



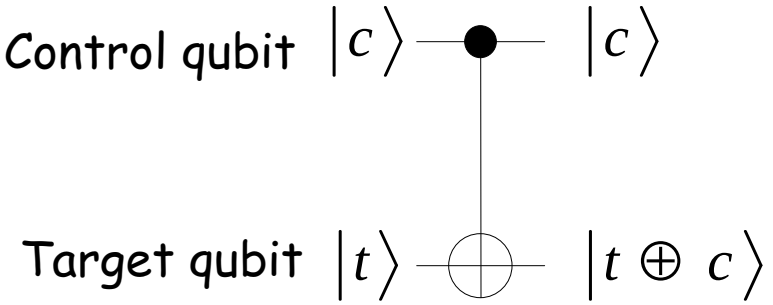
$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}; \quad H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}; \quad H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Phase gate:



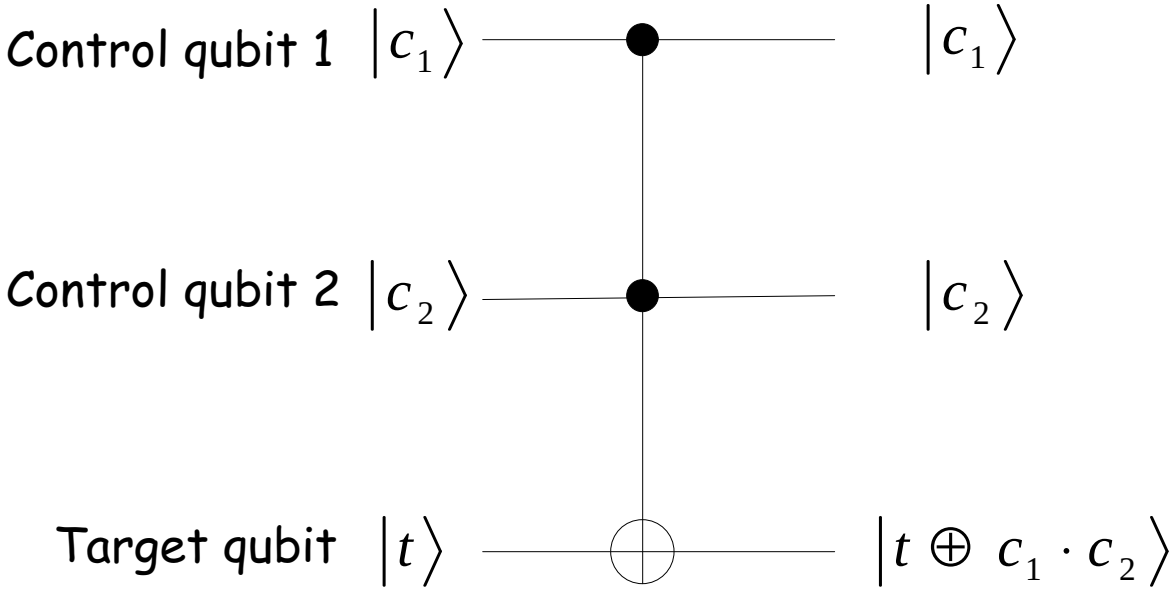
$$P|0\rangle = |0\rangle; \quad P|1\rangle = i|1\rangle$$

Controlled not gate:



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Toffoli gate



Universal Logic Gates

Suppose U is an arbitrary unitary transformation on n qubits.

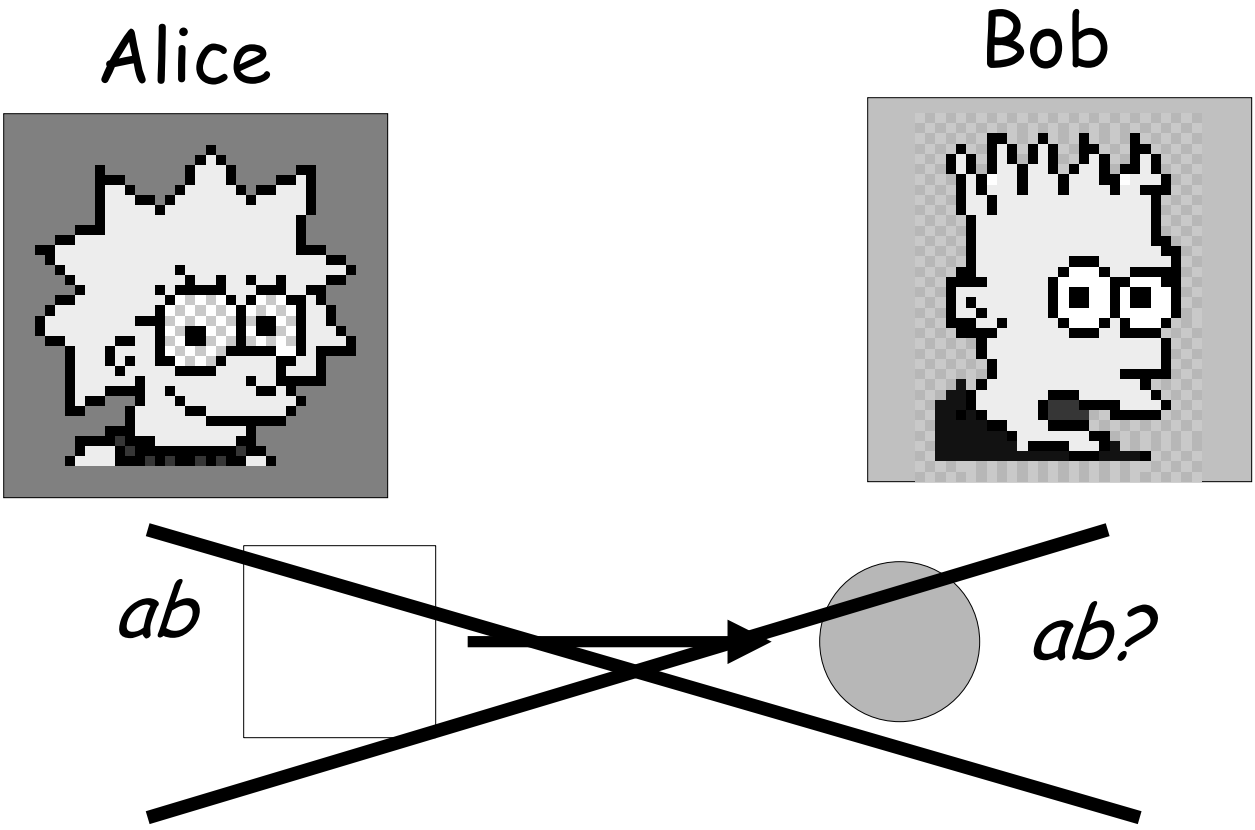
U can be approximated arbitrarily well by a sequence of Hadamard gates, phase gates, controlled not gates, and Toffoli gates.

These gates may all be performed fault-tolerantly, so in principle noise is not a problem.

Quantum circuit model

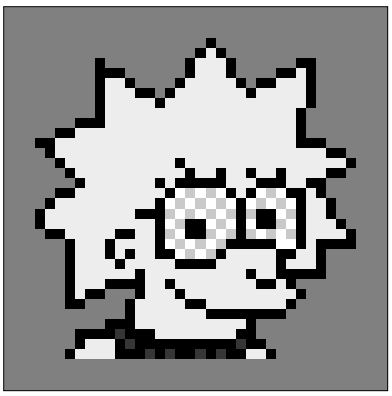
1. Prepare state $|0,0,\ldots,0\rangle$
2. Apply circuit of Hadamard, phase, controlled not and Toffoli gates.
3. Measure in the computational basis.

Superdense coding

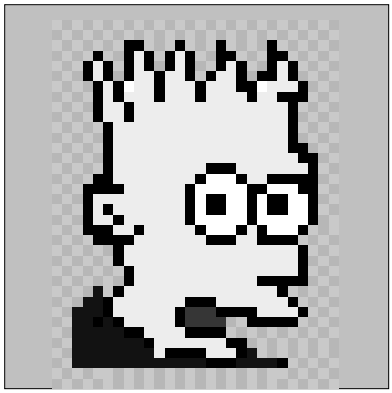


Superdense coding

Alice



Bob



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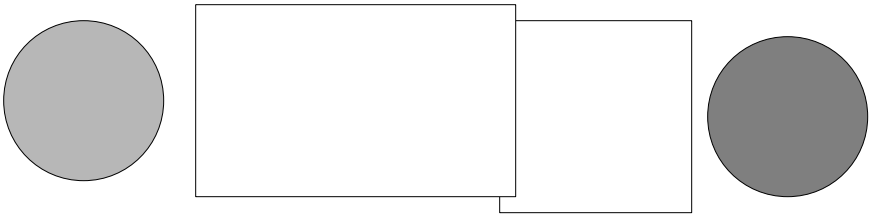

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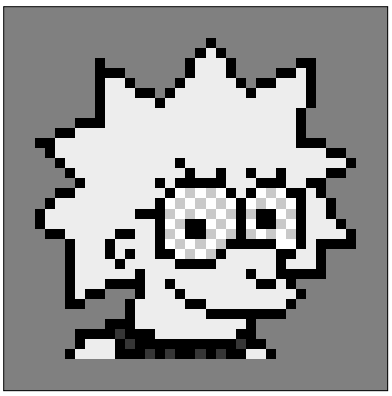

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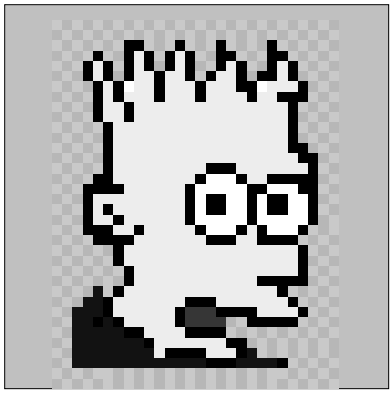

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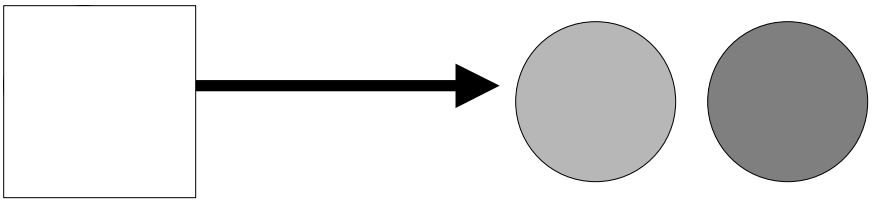

ab?

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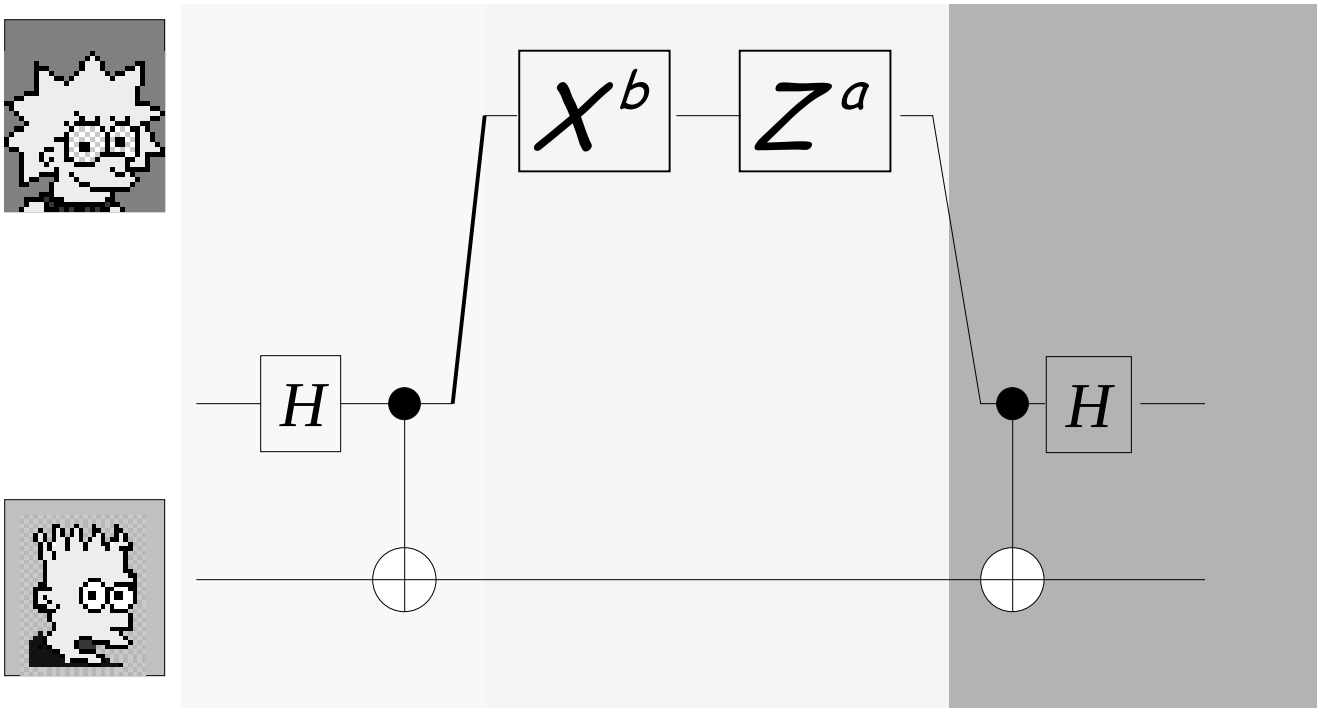


$$00 \rightarrow 00 + 10 \rightarrow 00 + 11$$

$$00 + 11 \quad (b = 0) \qquad 10 + 01 \quad (b = 1)$$

$$00 + 11 \quad (a = 0) \qquad 00 - 11 \quad (a = 1) \qquad 10 + 01 \quad (a = 0) \qquad -10 + 01 \quad (a = 1)$$

$$ab = 01: \; 10 + 01 \rightarrow 11 + 01 \rightarrow 01$$

Example: Deutsch's problem

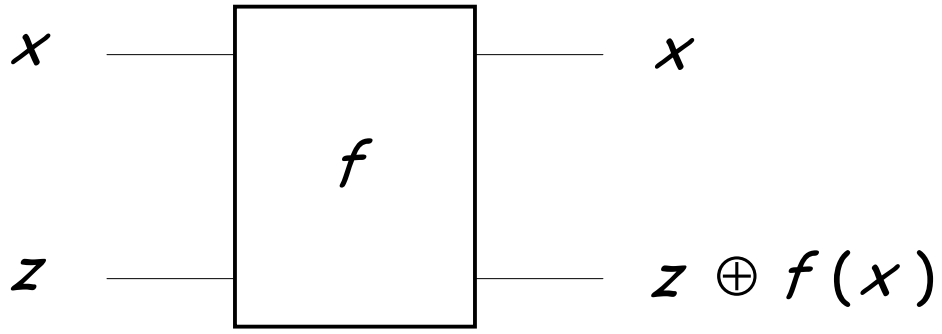
Given a black box computing a function $f : \{0,1\} \to \{0,1\}$

Our task is to determine whether f is constant or balanced?

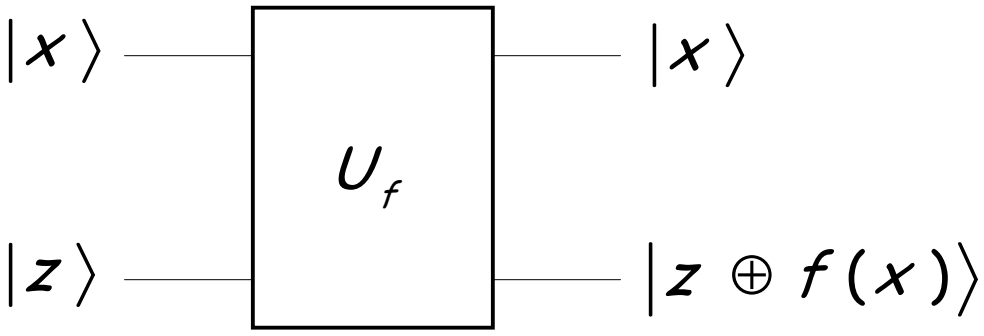
Classically we need to evaluate both $f(0)$ and $f(1)$.

Quantumly we need only use the black box for $f(\bullet)$ once!

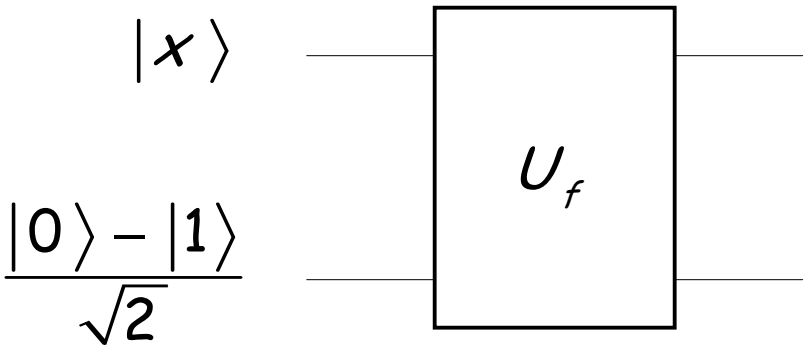
Classical black box



Quantum black box



Putting information in the phase



$f(x) = 0$:

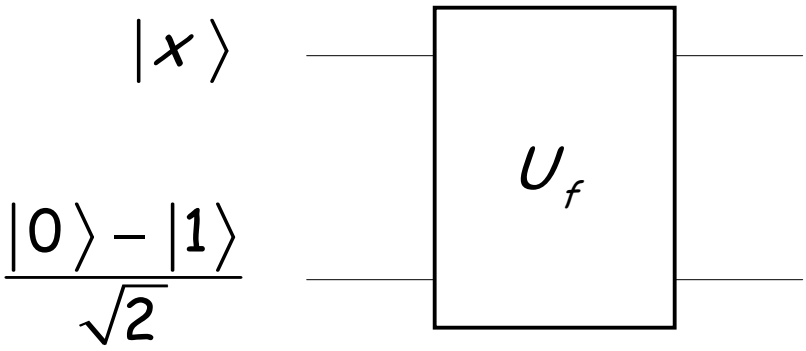
$$|x\rangle(|0\rangle - |1\rangle) \rightarrow |x\rangle(|0\rangle - |1\rangle)$$

$f(x) = 1$:

$$|x\rangle(|0\rangle - |1\rangle) \rightarrow |x\rangle(|1\rangle - |0\rangle)$$
$$= -|x\rangle(|0\rangle - |1\rangle)$$

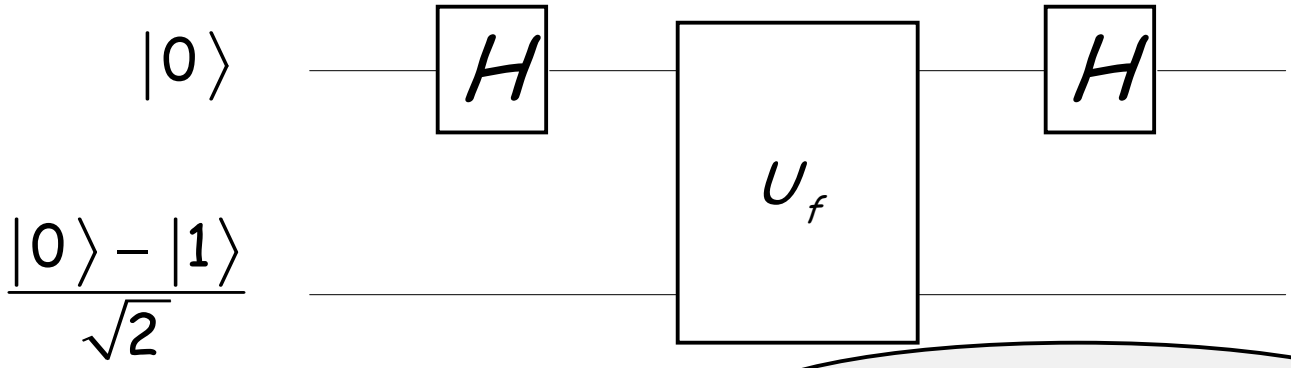
$$|x\rangle(|0\rangle - |1\rangle) \rightarrow (-1)^{f(x)}|x\rangle(|0\rangle - |1\rangle)$$

Putting information in the phase



$$|x\rangle \rightarrow (-1)^{f(x)} |x\rangle$$

Quantum algorithm for Deutsch's problem



Quantum parallelism

$$|0\rangle \rightarrow |0\rangle + |1\rangle$$

$$\rightarrow (-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle$$

$$\rightarrow (-1)^{f(0)}(|0\rangle + |1\rangle) + (-1)^{f(1)}(|0\rangle - |1\rangle)$$

$$= \left[(-1)^{f(0)} + (-1)^{f(1)}\right]|0\rangle + \left[(-1)^{f(0)} - (-1)^{f(1)}\right]|1\rangle$$

f const. $\Rightarrow$ all amplitude in $|0\rangle$.

f balan. $\Rightarrow$ all amplitude in $|1\rangle$.